كلية الهندسة بشبرا
FACULTY OF ENGINEERING AT SHOUBRA

Electrical & Magnetic Fields
Electromagnetic Fields / Fundamentals
(ELE222)(ELE242)(CCE302)

Lecture (04)
Coulomb's Force & Electric Field Intensity

Assoc. Prof. Dr/ Moataz M. Elsherbini
motaz.ali@feng.bu.edu.eg

2.2 Coulomb's Law

2.2.1 Electrostatic Force for two Charges

- Coulomb's law relates the force between two charged bodies which are very small in size compared to their separation. Ideally, the charged bodies must be so small that they can be considered as 'point charges'.
- Coulomb's law states that: .

1. Like charges repel where as unlike charges attract.
2. The magnitude of the force is proportional to the product of the magnitudes of the charges.
3. The magnitude of the force is inversely proportional to the square of the distance between the charges.
4. The direction of the force is along the line joining the charges .
5. The force depends upon the medium in which the charges are placed.

- If two point charges Q_1 and Q_2 [measured in coulombs (C)] are considered to be separated by a distance r [m] as shown in Fig 2.2, then according to Coulomb's law in the free space the force between these charges is:

$$\vec{F} = K_e \frac{Q_1 Q_2}{r^2} \vec{a}_r = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2} \hat{a}_r \qquad (2.2)$$

where K_e is the constant of proportionality and equals to $1/4\pi\varepsilon_o$, The quantity ε_o is the permittivity of free space [ε_o= 8.854 $\times 10^{-12}$ = $(1/36\pi)\times 10^{-9}$ Farad/m] and is the unit $\hat{a}_r$ vector along the line of Q_1 and Q_2.

2.2 Coulomb's Law (Continued)

2.2.1 Electrostatic Force for two Charges

- The permittivity is conveniently expressed as the product of the permittivity ε_o of vacuum multiplied by a dimensionless number ε_r, called the relative permittivity. Thus:
  $\varepsilon = \varepsilon_r\ \varepsilon_o$
  where ε_r is the relative permittivity and ε_o = permittivity of vacuum [ε_o= 8.854 x10^{-12} = (1/36π)x10^{-9} F/m] and $\varepsilon_r = 1$. For a dielectric material ε_r > I.
- For air at atmospheric pressure ε_r =1.0006. This differs so little from vacuum [ε_r =1 exactly] that for most practical situations ε_o= 8.854 x10^{-12} = (1/36π)x10^{-9} F/m is taken as the permittivity of both air and vacuum.

Example 1 Two charges Q_1 = 1.5 μC and Q_2 = -2 μC are placed 8 cm apart. Calculate the magnitude and direction of the Coulomb force.

Solution Apply (2.2), we get:

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}\hat{a}_r$$

$$= \frac{1.5x10^{-6}x(-2x10^{-6})}{4\pi x 8.85x10^{-12}x(0.08)^2}\hat{a}_r$$

$$= -4.21\ \hat{a}_r\ \ N$$

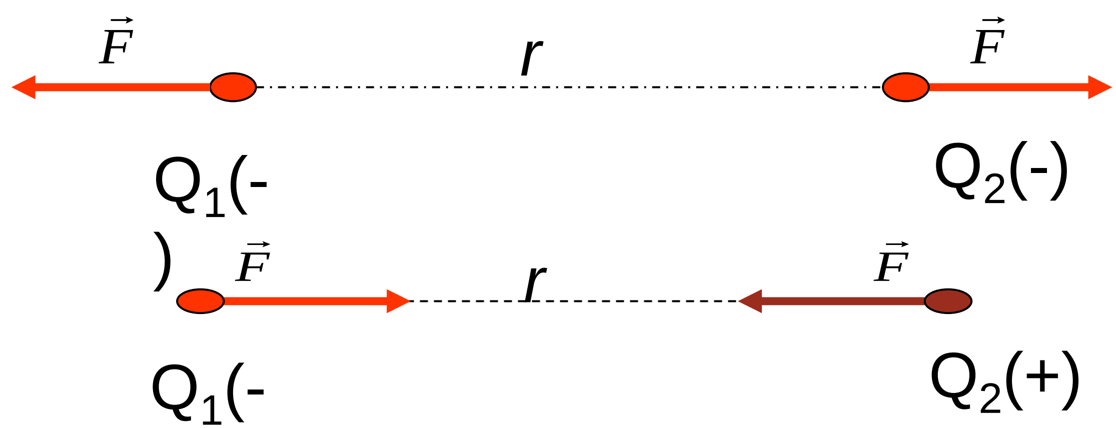


Fig. 2.2 Force on two charges $Q_1$and Q_2

Example 2

A 2mc positive charge is isolated in a vacuum at $P_1(3,-2,-4)$ and 5μc negative charge at $P_2(1,-4,2)$

(a) Find the vector force on negative charge

(b) " " magnitude of Force on charge at P1

Sol

$\overline{F_{12}} = 9\times10^9 \times \frac{q_1 q_2}{r^2}\, \overline{a_{R12}}$

$= 9\times10^9 \times \frac{(2\times10^{-3})(-5\times10^{-3})}{r^2}\, \overline{a_{R12}}$ $\qquad r = |\overline{R_{12}}|$

$\overline{R_{12}} = \overline{R_2} - \overline{R_1} = (1,-4,2) - (3,-2,-4)$

$= (-2,-2,6) = -2\hat{a_x} - 2\hat{a_y} + 6\hat{a_z}$

$|\overline{R_{12}}| = r = \sqrt{(-2)^2+(-2)^2+(6)^2} = \sqrt{44}$

$\therefore \overline{a_R} = \frac{\overline{R_{12}}}{|\overline{R_{12}}|}$

$\therefore \overline{F_{12}} = \frac{9\times10^9 \times 2\times10^{-3} \times -5\times10^{-3}}{(\sqrt{44})^2} \cdot \frac{(-2\hat{a_x} - 2\hat{a_y} + 6\hat{a_z})}{\sqrt{44}}$

$\overline{F_{12}} = 0.616\,\hat{a_x} + 0.616\,\hat{a_y} - 1.84\,\hat{a_z}$

(b) Magnitude $= |\overline{F_{12}}| = \sqrt{(0.616)^2 + (0.616)^2 + (-1.84)^2}$

$= 2.04\ N$

Example 3

* Find total Force experience (exerts) on 7 μc (2D) ندرس الأول

Sol/

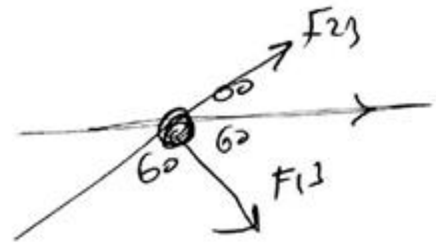
$$|F_{13}| = \frac{9\times10^{9}(4\times10^{-6})(7\times10^{-6})}{(1)^2} \rightarrow$$ الاتجاه بالرسم $= 252\times10^{3}$

الرسم عند اشحنا لا وزاحب القوى عندها
وبما ان الاولى والثانية موجبة تنافر

$$|F_{23}| = \frac{(2\times10^{-6})(7\times10^{-6})}{(1)^2} \rightarrow 126\times10^{3}$$

→ 3 & 1 تجاذب
3 & 2 تنافر

نحسب المحاور أ



المثلث متساوي الأضلاع زاويته 60°
و كل القوى بعدين

$$\therefore \Sigma F_x = F_{13}\cos 60 + F_{23}\cos 60 = 189\times10^{3}\ \text{N}$$

$$\Sigma F_y = F_{23}\sin 60 - F_{13}\sin 60 = 189\sqrt{3}\times10^{3}\ \text{N}$$

$$\bar{F}_{on\,3} = 189\times10^{3}\,\hat{a}_x + 189\sqrt{3}\times10^{3}\,\hat{a}_y$$

2.2 Coulomb's Law (Continued)

2.2.1 Electrostatic Force for two Charges

- If the location of point charges Q_l and Q_2 are represented by their position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively, as shown in Fig. 2.3, then the Coulomb's law equation (2.2) is modified as:

$$\vec{F}_{BA} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\hat{a}_{RBA} \quad (2.3a)$$

or

$$\vec{F}_{AB} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\hat{a}_{RAB} \quad (2.3b)$$

Where $\hat{a}_{RBA}$ and $\hat{a}_{RAB}$ are the unit vectors which are given as follows:

$$\hat{a}_{RAB} = \frac{\vec{R}_{AB}}{|\vec{R}_{AB}|} = \frac{\vec{r}_B - \vec{r}_A}{R} \quad (2.4a)$$

and

$$\hat{a}_{RBA} = \frac{\vec{R}_{BA}}{|\vec{R}_{BA}|} = \frac{\vec{r}_A - \vec{r}_B}{R} \quad (2.4b$$

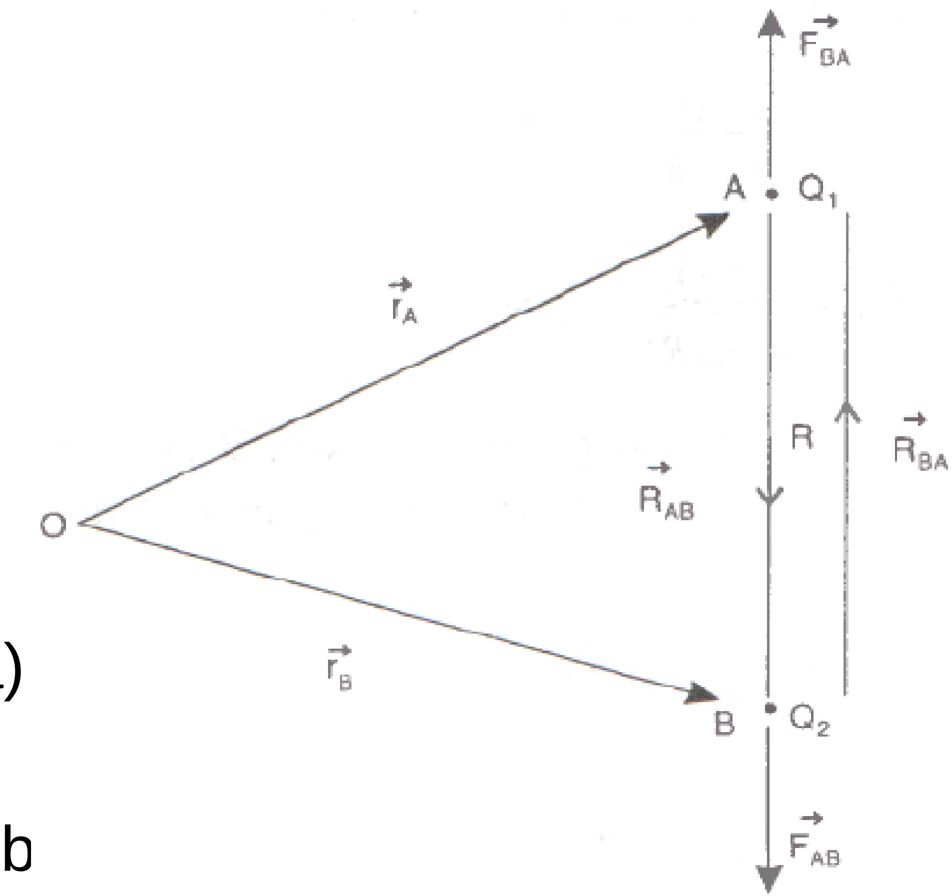


Fig. 2.3 Force on charges Q_1 and Q_2

2.2 Coulomb's Law (Continued)

2.2.1 Electrostatic Force for two Charges

- From eqn. (2.4) to eqn. (2.3), we get:

$$\vec{F}_{BA} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^3}\vec{R}_{BA} \quad (2.5a) \qquad \vec{F}_{AB} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^3}\vec{R}_{AB} \quad (2.5b)$$

Example 4

Find the force on a charge Q_1= 20 μC, placed at the point (0, l, 2) m due to charge Q_2= -300 μC, placed at the point (2, 0, 0) m.

Solution Apply (2.4) and (2.5), we have:

$$\vec{R}_{BA} = \vec{r}_A - \vec{r}_B = (0,\ 1,\ 2) - (2,\ 0,\ 0) = (-2,\ 1,\ 2)$$

$$R = \sqrt{(-2)^2 + (1)^2 + (2)^2} = \sqrt{9} = 3$$

$$\vec{F}_{BA} = \frac{20x10^{-6}\, x(-300x10^{-6})}{4\pi x 8.85x10^{-12}\, x3^3} x(-2,\ 1,\ 2)$$

$$= -1.99x(-2\hat{a}_x + \hat{a}_y + 2\hat{a}_z)$$

$$\vec{F}_{BA} = 3.98\hat{a}_x - 1.99\hat{a}_y - 3.98\hat{a}_z)$$

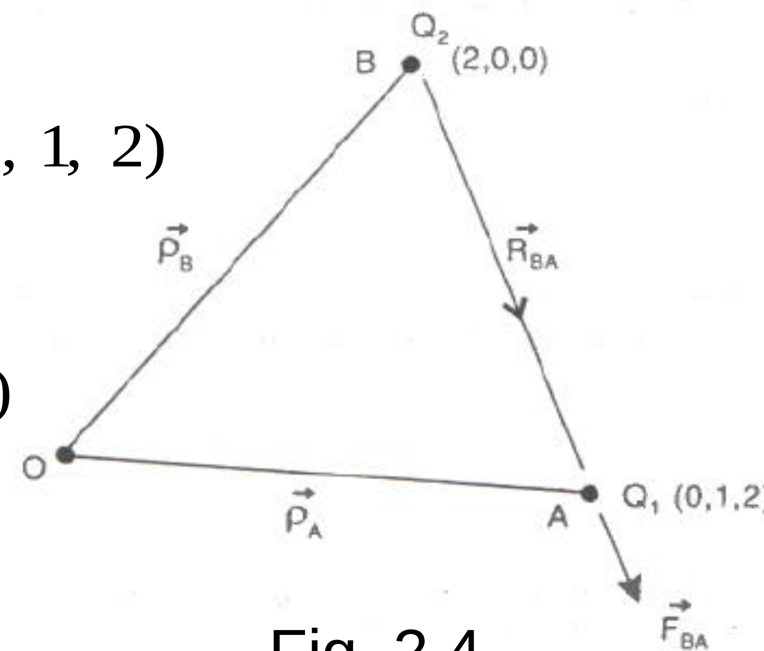


Fig. 2.4

2.2 Coulomb's Law (Continued)

2.2.2 Electrostatic Forces for Systems with more than two Charges

- If there are several point charges Q_1, Q_2, Q_3, ..., Q_n located at different points as shown in Fig. 2.5, then by using superposition principle, the force $\vec{F}$ experienced by a test charge situated at a point P is the vector sum of forces experienced by the test charge due to individual charges.

$$\vec{F} = \frac{Q_1 q}{4\pi\varepsilon_o R_1^2}\hat{a}_{R_1} + \frac{Q_2 q}{4\pi\varepsilon_o R_2^2}\hat{a}_{R_2} + \frac{Q_3 q}{4\pi\varepsilon_o R_3^2}\hat{a}_{R3} + ... + \frac{Q_n q}{4\pi\varepsilon_o R_n^2}\hat{a}_{R_n} \quad (2.6a)$$

$$\vec{F} = \sum_{j=1}^{n} \frac{Q_j q}{4\pi\varepsilon_o R_j^2}\hat{a}_{R_j} \quad (2.6b)$$

- Similarly, if there are n charges Q_1, Q_2, Q_3, ..., Q_n located at points with position vectors $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_n$... respectively, then the resultant force $\vec{F}$ on a test charge q at a point with the position vector $\vec{r}$ is the vector sum of the forces exerted on the test charge 'q' by all the other charges individually (2.6) becomes:

$$\vec{F} = \frac{Q_1 q}{4\pi\varepsilon_o |\vec{r} - \vec{r}_1|^3}.(\vec{r} - \vec{r}_1) + \frac{Q_2 q}{4\pi\varepsilon_o |\vec{r} - \vec{r}_2|^3}.(\vec{r} - \vec{r}_2) + ... + \frac{Q_n q}{4\pi\varepsilon_o |\vec{r} - \vec{r}_n|^3}.(\vec{r} - \vec{r}_n) \quad (2.7a)$$

$$\vec{F} = \frac{q}{4\pi\varepsilon_o}\sum_{j=1}^{n} \frac{Q_j(\vec{r} - \vec{r}_2)}{|\vec{r} - \vec{r}_j|^3} \quad (2.7b)$$

2.2 Coulomb's Law (Continued)

2.2.2 Electrostatic Forces for Systems with more than two Charges

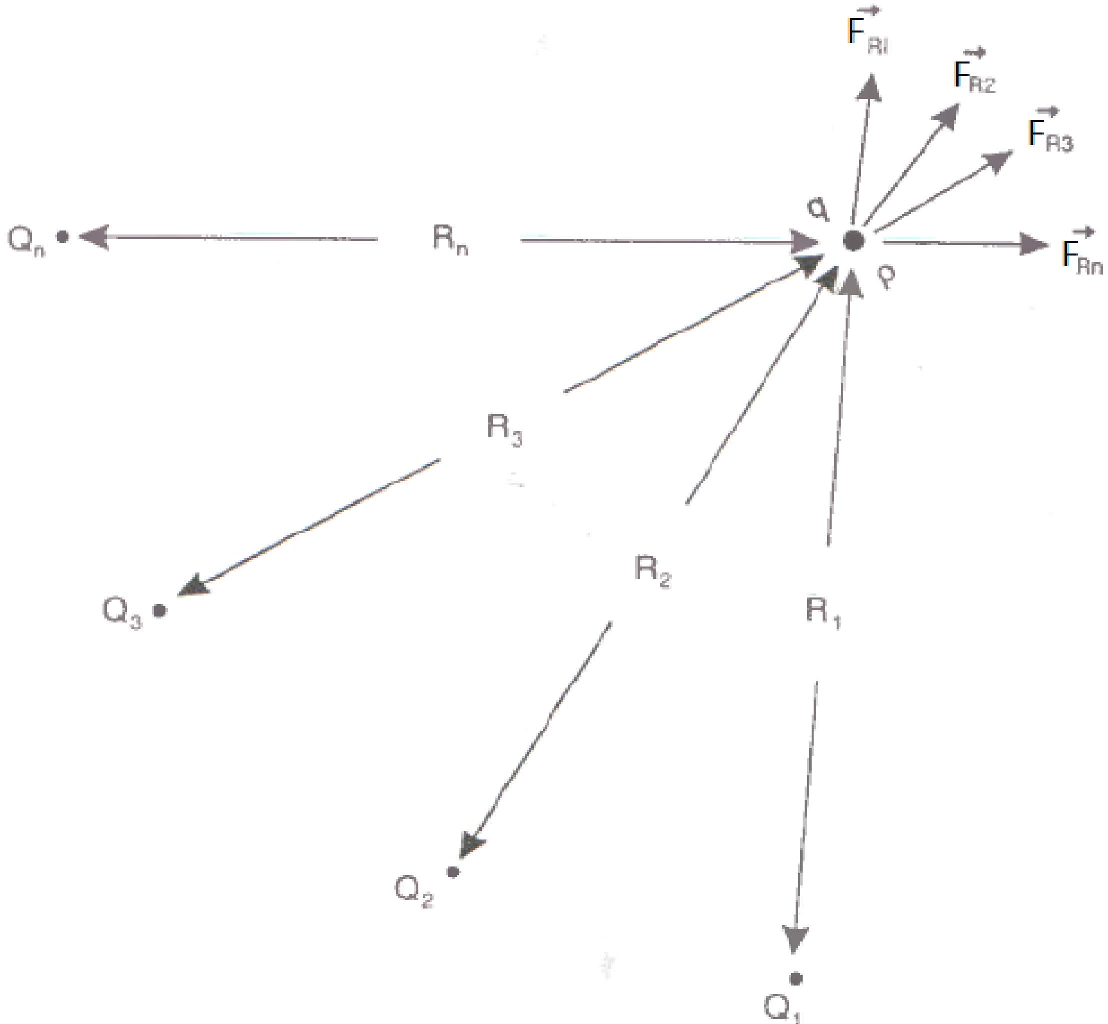
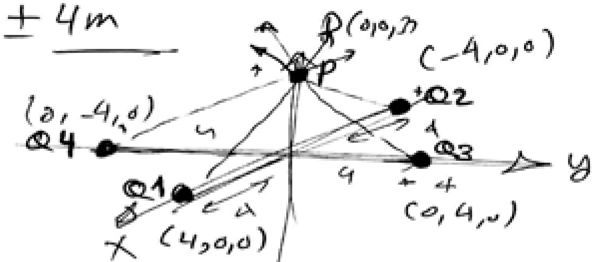


Fig. 2.5 Force at point P due to assembly of charges.

Example 5

Find Force on 100 μC charge at (0,0,3) m of Four like charges of 20 μC are located on x and y axis at ± 4m



sol

$$\vec{F}_t = \frac{Q_1 Q_p}{r_1^2} \times 9\times10^9 \hat{a}_{R_1} + \frac{Q_2 Q_p}{r_2^2} \times 9\times10^9 \hat{a}_{R_2} + \frac{Q_3 Q_p}{r_3^2} \times 9\times10^9 \hat{a}_{R_3} + \frac{Q_4 Q_p}{r_4^2} \times 9\times10^9 \hat{a}_{R_4}$$

$\because Q_1 = Q_2 = Q_3 = Q_4 = 20\times10^{-6}$

$$\therefore \vec{F}_t = 20\times10^{-6} \times 100\times10^{-6} \times 9\times10^9 \left[\frac{\hat{a}_{R_1}}{r_1^2} + \frac{\hat{a}_{R_2}}{r_2^2} + \frac{\hat{a}_{R_3}}{r_3^3} + \frac{\hat{a}_{R_4}}{r_4^2} \right]$$

* $\vec{R}_1 = \vec{R}_p - \vec{R}_1 = (-4, 0, +3) = -4\hat{a}_x + 3\hat{a}_z$

  $|R_1| = r_1 = \sqrt{(-4)^2 + (3)^2} = 5$ & $\hat{a}_{R_1} = \left(-\frac{4}{5}\hat{a}_x + \frac{3}{5}\hat{a}_z\right)$, I

* $\vec{R}_2 = \vec{R}_p - \vec{R}_2 = (4, 0, 3) = 4\hat{a}_x + 3\hat{a}_z$

  $r_2 = 5$ & $\hat{a}_{R_2} = \frac{4}{5}\hat{a}_x + \frac{3}{5}\hat{a}_z$ → II

* $\vec{R}_3 = \vec{R}_p - \vec{R}_3 = (0, -4, 3) = -4\hat{a}_y + 3\hat{a}_z$

  $r_3 = 5$ & $\hat{a}_{R_3} = -\frac{4}{5}\hat{a}_y + \frac{3}{5}\hat{a}_z$ → III

* $\vec{R}_4 = \vec{R}_p - \vec{R}_4 = (0, 4, 3) = 4\hat{a}_y + 3\hat{a}_z$

  $r_4 = 5$ & $\hat{a}_{R_4} = \frac{4}{5}\hat{a}_y + \frac{3}{5}\hat{a}_z$ → IV

$$F_t = \frac{9\times10^9 \times 20\times10^{-6} \times 100\times10^{-6}}{5^2} \left[4 \times \frac{3}{5}\hat{a}_z\right] = 1.73\,\hat{a}_z \quad (N)$$

2.2 Coulomb's Law (Continued)

2.2.2 Electrostatic Forces Summary

- Interaction between static charges is defined by Coulomb law.
- Electrostatic force is proportional to the charges and inversely proportional to charge separation (inverse square law).
- Direction of electrostatic force is defined by the polarity of charges.
- Force between a number of charges is found using principle of superposition.
- The charge carried by an electron is (-*e*) and that of proton is (+*e*) where [$e = 1.602 \times 10^{-19}$ C].
- Although the charge is very small, the electrostatic forces in solids are responsible for their strength under compression.
- Electrostatic phenomena are used in electrostatic copiers, paint sprays and can lead to explosions in oil tankers and need to be considered when handling metal-oxide semiconductor circuits.

- Coulomb's law is seldom (possibly never!) used in practise. Why?

  - Charges do not appear isolated.
  - To obtain a force due to many charges involves a vector summation – messy!
  - Distribution of charges is not always known.
- This leads to introduce a new quantity that can be used to more effectively solve problems which is the **electric field** $\vec{E}$**.**

2.2 Coulomb's Law (Continued)

2.2.3 Electric Field Intensity

- Dividing (2.2) by Q_2 gives a force per unit charge which is defined as the electric field $\vec{E}$. Its unit is Newton per Coulomb [N/C] or Volts per meter [V/m] .
- The concept is: charge Q_1 sets in its vicinity an influence such that any charge Q_2 can experience a Force $\vec{F}$. The influence is termed electric field, $\vec{E}$ which can be written as:.

$$\vec{E} = \frac{\vec{F}}{Q_2} = \frac{Q_1}{4\pi\varepsilon_o r^2}\hat{a}_r \;\; N/C \;\; or \;\; V/m \qquad (2.8)$$

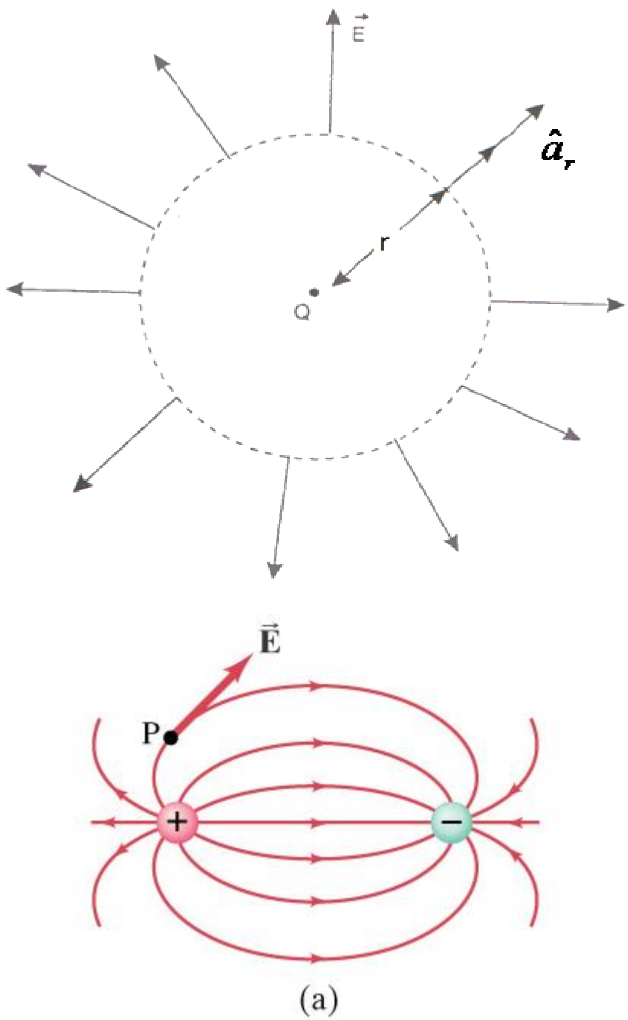
where r is the distance of Q_2 from Q_1. In general, electric field intensity at any point due to a point charge of Q coulomb is given by:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_o r^2}\hat{a}_r \;\; N/C \;\; or \;\; V/m \qquad (2.9)$$

- where r is the distance from the point charge to the point at which the field intensity is to be computed and $\vec{a}_r$ is the unit vector along the direction joining of the line the two points under consideration and directed away from the point charge. The electric field intensity of a point charge is thus directed everywhere radially away from the point charge, and on any spherical surface centered at the point charge its magnitude is constant as shown in Fig. 2.7.

2.2 Coulomb's Law (Continued)

2.2.3 Electric Field Intensity



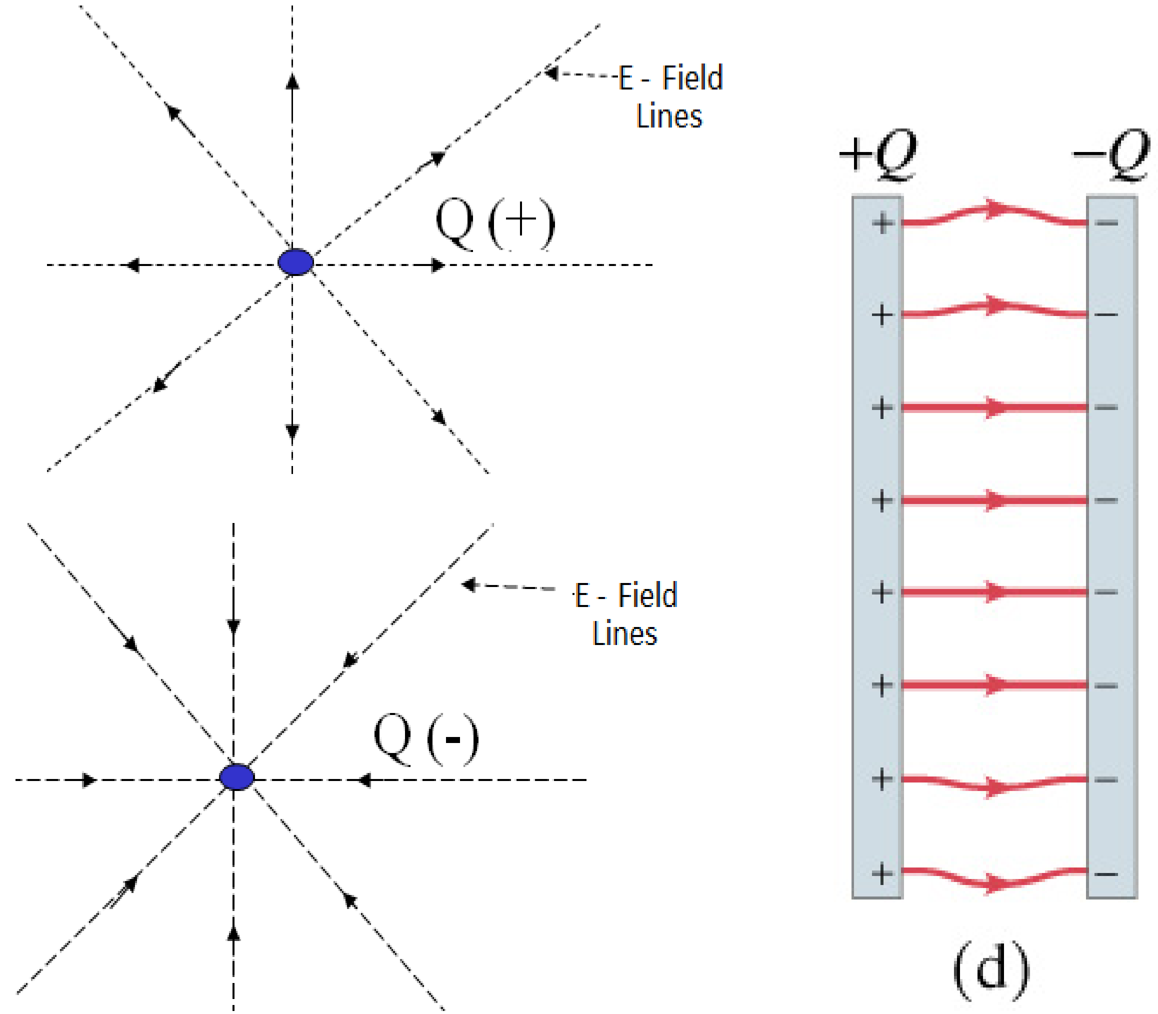


Fig. 2.7 Electric field of a point charge Q.

2.2 Coulomb's Law (Continued)

2.2.3 Electric Field Intensity

- If there are several point charges Q_1, Q_2, Q_3, ..., Q_n located at different points as shown in Fig. 2.5, then by using superposition principle, the electric field intensity on a test charge situated at a point P is given by:

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_o R_1^2}\hat{a}_{R_1} + \frac{Q_2}{4\pi\varepsilon_o R_2^2}\hat{a}_{R_2} + \frac{Q_3}{4\pi\varepsilon_o R_3^2}\hat{a}_{R3} + ... + \frac{Q_n}{4\pi\varepsilon_o R_n^2}\hat{a}_{R_n} \quad (2.10a)$$

$$\vec{F} = \sum_{j=1}^{n} \frac{Q_j}{4\pi\varepsilon_o R_j^2}\hat{a}_{R_j} \;\; V/m \quad (2.10b)$$

- Similarly, if there are n charges Q_1, Q_2, Q_3, ..., Q_n located at points with position vectors $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_n$... respectively, then the resultant electric field intensity $\vec{E}$ on a test charge at a point with the position vector $\vec{r}$ is the vector sum of the electric field intensity all the other charges individually (2.10) becomes:

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_o |\vec{r}-\vec{r}_1|^3}.(\vec{r}-\vec{r}_1) + \frac{Q_2}{4\pi\varepsilon_o |\vec{r}-\vec{r}_2|^3}.(\vec{r}-\vec{r}_2) + ... + \frac{Q_n}{4\pi\varepsilon_o |\vec{r}-\vec{r}_n|^3}.(\vec{r}-\vec{r}_n) \quad (2.11a)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\sum_{j=1}^{n} \frac{Q_j(\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_j|^3} \quad (2.11b)$$

2.2 Coulomb's Law (Continued)

2.2.3 Electric Field Intensity

Example 6: Four charges $Q_1 = Q_2 = Q_3 = Q_4 = 3$ pC are located at the comers of a 1-m square. The two charges an the left side of the square are positive. The two charges on the right side are negative. Find the electric field intensity $\vec{E}$ at the center of the square.

Solution: Draw a sketch (Fig. 2.8). The sketch solves the problem geometrically:

$$\vec{E}_{13} = \vec{E}_1 + \vec{E}_3 = \frac{2Q}{4\pi\varepsilon_o r^2}\hat{a}_{13} = \frac{2x3x10^{-12}}{4\pi x 8.85x10^{-12}x(0.707)^2}\hat{a}_{13} = 0.108\ \hat{a}_{13} V/m$$

$$\vec{E}_{24} = \vec{E}_2 + \vec{E}_4 = \frac{2Q}{4\pi\varepsilon_o r^2}\hat{a}_{24} = \frac{2x3x10^{-12}}{4\pi x 8.85x10^{-12}x(0.707)^2}\hat{a}_{24}$$

$$= 0.108\ \hat{a}_{24} V/m$$

From the Fig. 2.8, the x and y components of $\vec{E}$ are:

$$\vec{E}_x = [|\vec{E}_{13}|\cos\theta + |\vec{E}_{24}|\cos\theta]\hat{a}_x = 2x0.108\cos 45^o\,\hat{a}_x$$

$$= 153\ \hat{a}_x\ mV/m$$

$$\vec{E}_y = [-|\vec{E}_{13}|\sin\theta + |\vec{E}_{24}|\sin\theta]\hat{a}_y = 0$$

Therefore, $\vec{E} = 153\ \hat{a}_x\ mV/m$

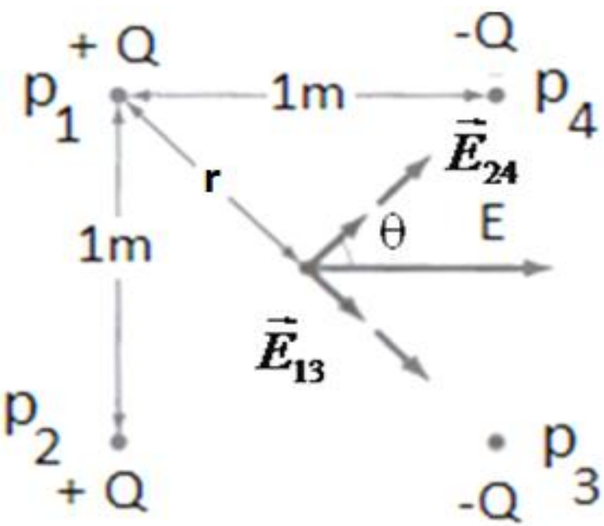
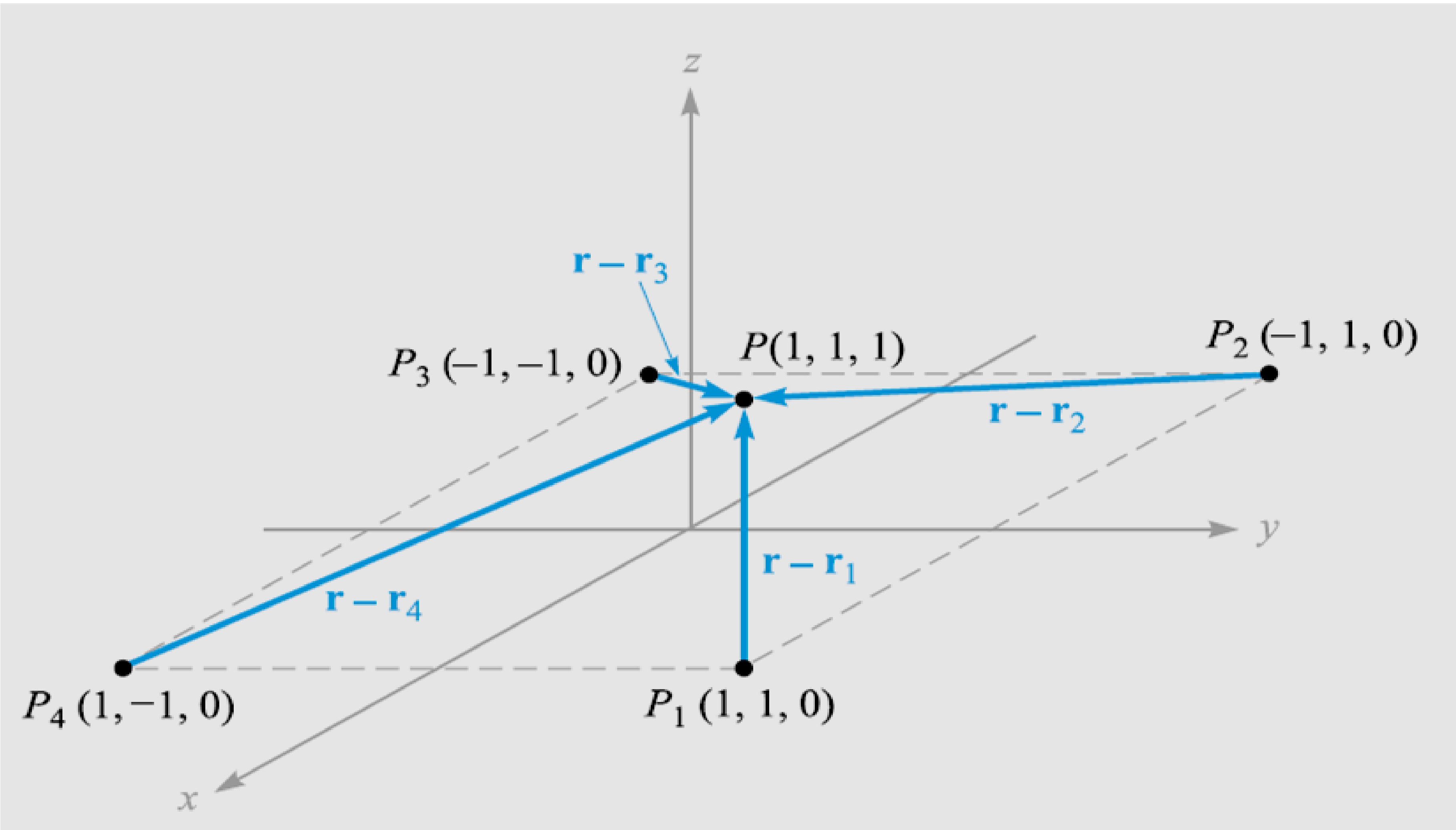


Fig. 2.8 System of four charges.

EXAMPLE (7): Find E at *P(1, 1, 1) caused by four identical* 3-nC charges located at *P1(1, 1, 0), P2(−1, 1, 0), P3(−1,−1, 0),* and *P4(1,−1, 0)*



$$\bar{r} = \hat{a}_x + \hat{a}_y + \hat{a}_z$$

$$\bar{r}_1 = \hat{a}_x + \hat{a}_y \qquad \therefore \bar{r} - \bar{r}_1 = \hat{a}_z \qquad |\bar{r} - \bar{r}_1| = \sqrt{(1)^2} = 1$$

$$\bar{r}_2 = -\hat{a}_x + \hat{a}_y \qquad \therefore \bar{r} - \bar{r}_2 = 2\hat{a}_x + \hat{a}_z \qquad |\bar{r} - \bar{r}_2| = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$$

$$\bar{r}_3 = -\hat{a}_x - \hat{a}_y \qquad \therefore \bar{r} - \bar{r}_3 = 2\hat{a}_x + 2\hat{a}_y + \hat{a}_z \qquad |\bar{r} - \bar{r}_3| = \sqrt{(2)^2 + (2)^2 + (1)^2} = 3$$

$$\bar{r}_4 = \hat{a}_x - \hat{a}_y \qquad \therefore \bar{r} - \bar{r}_4 = 2\hat{a}_y + \hat{a}_z \qquad |\bar{r} - \bar{r}_4| = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$$

$$\frac{Q}{4\pi\varepsilon_o} = \frac{3x10^{-9}}{4\pi x 8.854 x 10^{-12}} = 26.96$$

$$So, \quad \bar{E} = 26.96\left[\frac{\hat{a}_z}{1}\frac{1}{1^2} + \frac{2\hat{a}_x + \hat{a}_z}{\sqrt{5}}\frac{1}{(\sqrt{5})^2} + \frac{2\hat{a}_x + 2\hat{a}_y + \hat{a}_z}{3}\frac{1}{3^2} + \frac{2\hat{a}_y + \hat{a}_z}{\sqrt{5}}\frac{1}{(\sqrt{5})^2}\right]$$

$$\bar{E} = 6.82\hat{a}_x + 6.82\hat{a}_y + 32.8\hat{a}_z \qquad V/m$$

Thank you for your attention

Dr. Moataz Elsherbini